PR 502: Robot Dynamics & Control

Robot Kinematics for Orientation

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Forward and Inverse Kinematics for orientation

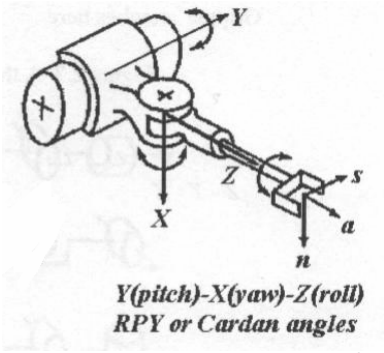
- If the moving frame attached to the hand of the robot has already moved to the desired position but the required orientation is not achieved:
  - Then the orientation must be changed until the desired orientation is achieved

Forward and Inverse Kinematics for orientation

- Three possible methods for orientation change;
  - Roll Pitch and Yaw (RPY) angles
  - Euler angles
  - Articulated joints

Roll Pitch and Yaw (RPY) angles (Cardan angles)

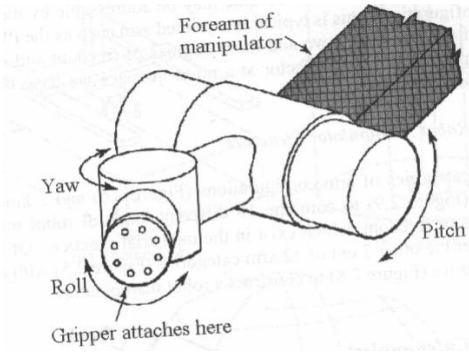
- Roll, Pitch and Yaw is basically a sequence of three rotations about Z, Y and X axis respectively.

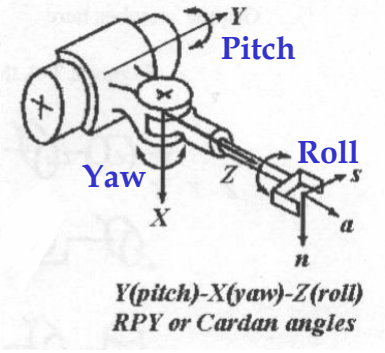


Y(pitch)-X(yaw)-Z(roll) RPY or Cardan angles

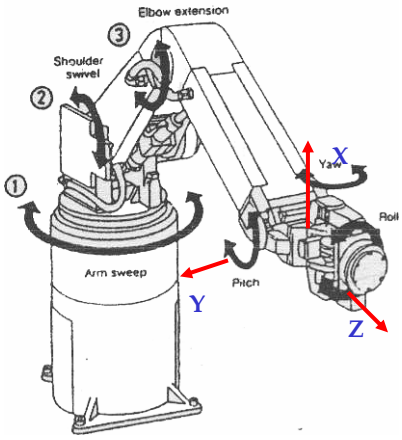
RPY Wrist configuration

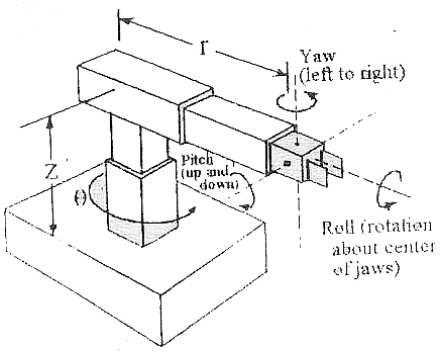
- RPY angles





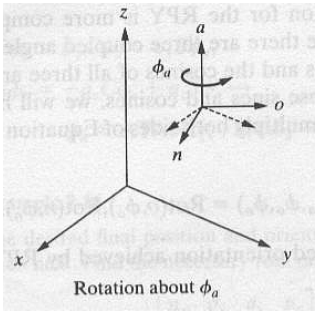
RPY Wrist configuration



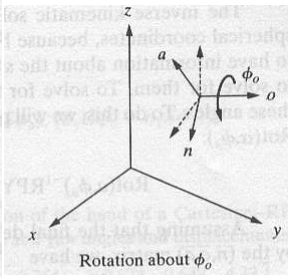


Kinematics of RPY Wrist

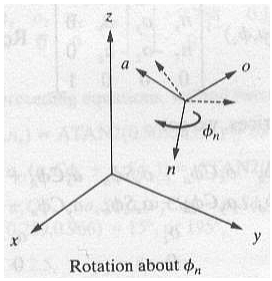
- RPY angles



Roll



Pitch



Yaw

Kinematics of RPY Wrist

- RPY angles

$$\mathrm{RPY}(\phi_a,\phi_o,\phi_n) = \mathrm{Rot}(a,\phi_a)\mathrm{Rot}(o,\phi_o),\mathrm{Rot}(n,\phi_n)$$

Substituting to corresponding matrices and multiplying together

$$\begin{bmatrix} C\phi_a C\phi_o & C\phi_a S\phi_o S\phi_n - S\phi_a C\phi_n & C\phi_a S\phi_o C\phi_n + S\phi_a S\phi_n & 0 \\ S\phi_a C\phi_o & S\phi_a S\phi_o S\phi_n + C\phi_a C\phi_n & S\phi_a S\phi_o C\phi_n - C\phi_a S\phi_n & 0 \\ -S\phi_o & C\phi_o S\phi_n & C\phi_o C\phi_n & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

C – cos, S - sin

Position is not changed

Kinematics of RPY Wrist

- Inverse kinematic equation
  - Inverse kinematic solution is complicated because sine and cosine values of all three angles have to be found
  - To solve for these sines and cosines, these angle have to be decoupled.
  - This can be achieved by pre-multiplying both sides of the equation by the inverse of Rot(a,Φ_a)

Complete Kinematic Model

- Representation of the frame at the end effector relative to the frame is the product of the two matrices representing the position change and the orientation change:
  - Ex: for spherical robot

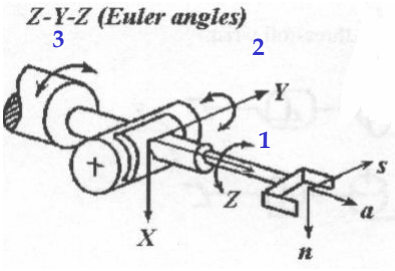
$$^{R}T_{H} = T_{sph}(r,\beta,\gamma)RPY(\phi_a,\phi_o,\phi_n)$$

$^{R}T_{H}$ Position and orientation of hand frame w.r.t. reference frame

T_{sph} Matrix for position change w.r.t. reference frame

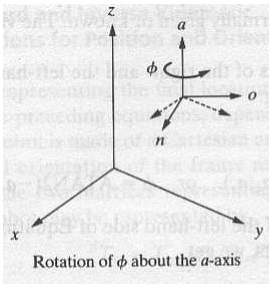
Euler angles

- The only difference between RPY angles and Euler angles in, in Euler angles the last rotation is made about Z axis instead of X axis

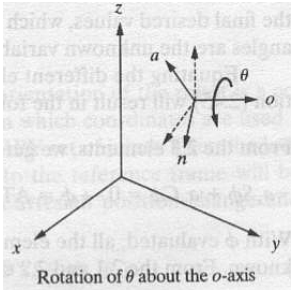


Euler angles

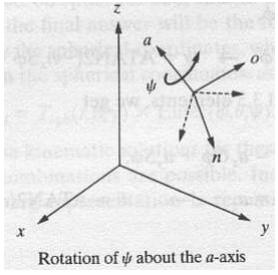
- Euler angles



Rotation of ϕ about the a-axis



Rotation of θ about the o-axis



Rotation of ψ about the a-axis

Wrist configurations

- Euler angles

$$\text{Euler}(\phi,\theta,\psi) = \text{Rot}(a,\phi)\text{Rot}(o,\theta),\text{Rot}(a,\psi)$$

$$\begin{bmatrix} C\phi C\theta C\psi - S\phi S\psi & -C\phi C\theta S\psi - S\phi C\psi & C\phi S\theta & 0 \\ S\phi C\theta C\psi + C\phi S\psi & -S\phi C\theta S\psi + C\phi C\psi & S\phi S\theta & 0 \\ -S\theta C\psi & S\theta S\psi & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Wrist configurations