IDENTIFY FAILURE AND REPAIR DISTRIBUTION

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The goal of system modeling

- The goal of system modeling is to provide quantitative forecasts of various system performance measures such as downtime, availability, number of failures, capacity, and cost. Evaluation of these measures is important to making optimal decisions when designing a system to either minimize overall cost or maximize a system performance measure within the allowable budget and other performance-based constraints.

Two important factors taken into account in the analysis of a system are

- failure behaviors
- repair behaviors of the system components.

Those are often defined in terms of distributions, or how the failures and repairs occur during the time period the system is operational.

Therefore, selecting the appropriate distributions for these failure and repair times is critical to analyzing your system metrics accurately.

Three step process

- Identify candidate distribution
- Estimating parameters
- Performing a goodness-of-fit test

In general, failure and repair times for systems and components are thought of as being random in nature.

So can be modeled using statistical distribution techniques widely recognized in the mathematical sciences.
A useful way to easily visualize the characteristics of a failure or repair distribution is through the use of

1. probability density functions (pdf).
   or
2. the cumulative distribution function (cdf)

This can be done manually or using Weibull analysis software tool

**Distributions are referred to as**

- single-parameter
  (only one variable is used to define the shape of the curve)
- two-parameter distributions.
  (require two variables to accurately model the distribution)

**Main types of the distributions**

- Exponential
- Normal
- Lognormal
- Weibull

**Construct a histogram**

- Grouping the failure times of the random sample into class and plotting the frequency of the number of observations within each class versus the interval time of each class
How to define number of classes

- Too many or too few classes tends to hide correct distribution
- Sturges’ rule
  \[ K = [1 + 3.3 \log(n)] \]
  where \( K \) = number of classes
  \( n \) = sample size

Exponential distribution

- If the failure rate is constant, which is generally true for electronic components during the main portion of their useful life, the reliability of the component follows an exponential distribution. The exponential distribution is a single-parameter distribution

- Exponential reliability function can be derived to be:
  \[ R(t) = 1 - \int_{0}^{t} \lambda e^{-\lambda t} \, dt = 1 - [1 - e^{-\lambda t}] = e^{-\lambda t} \]

  The exponential failure rate function is
  \[ \lambda(t) = \frac{d}{dt} R(t) = \lambda e^{-\lambda t} \]

  The exponential Mean-Time-To-Failure (MTTF) is given by
  \[ MTTF = \frac{1}{\lambda} \]

Normal

- The normal (or Gaussian) distribution is frequently used to describe equipment failure behavior that has increasing failure rates with time
- It is the best known two-parameter distribution, defined by the mean (\( \mu \)) and standard deviation (\( \sigma \)).
Normal distribution

- MTTF is the median
- The reliability for a mission of time $T$ is determined by
  \[ R(T) = \frac{1}{T} \int_0^T e^{-t/T} \, dt \]
- The instantaneous normal failure rate is given by
  \[ \lambda(T) = \frac{MTTF}{T} \left( \frac{1}{\sigma \sqrt{2\pi}} \right) e^{-\frac{(t-\mu)^2}{2\sigma^2}} \]

Lognormal distribution

- Lognormal distributions are encountered frequently in maintainability data (time to repair), chemical-process equipment failures and repairs, crack propagation, in probabilistic design
- Distribution parameters are the mean ($\mu$) and standard deviation ($\sigma$)

Weibull distribution

- Weibull distribution can model failures cause by fatigue, corrosion, mechanical abrasion, diffusion, and other degradation processes
- Characteristic life ($\eta$) and shape factor ($\beta$) values. (Beta determines the shape of the distribution)
- Two-parameter distribution

\[ f(t) = \begin{cases} \frac{\beta}{\eta} \left( \frac{t}{\eta} \right)^{\beta-1} e^{-\left( \frac{t}{\eta} \right)^\beta} & t > 0 \\ 0 & t \leq 0 \end{cases} \]

\[ \eta = \text{scale parameter}, \quad \beta = \text{shape parameter (or slope)}, \quad \gamma = \text{location parameter.} \]
**The Mean or MTTF**

\[ \bar{T} = \frac{1}{n+0.4} \sum_{i=1}^{n} T_i \]

**The Weibull Reliability Function**

\[ R(T) = e^{-\left( \frac{T}{\eta} \right)^\beta} \]

**The Weibull Failure Rate Function**

\[ r(T) = \frac{\beta}{\eta} \left( \frac{T}{\eta} \right)^{\beta-1} \]

*use of cumulative distribution function, or cdf,*

Here, cdf is use as

\[ F(t) = 1 - e^{-\left( \frac{t}{\eta} \right)^\beta} \]

Where \( i = 1, 2, \ldots, n \)

\( n = \text{sample size} \)

Proper fit distribution gives approximate straight line

**Exponential plot**

- Here plot \( \ln[1/1-F(t)] \) Vs \( T(i) \) on Exponential graph paper

- Since that gives straight line which discribe \( Y = a + bX \)
- \( 1/b = \text{MTTF} \)

**Weibull plot**

- Here plot \( \ln[1/1-F(t)] \) Vs \( \ln[T(i)] \) on Weibull graph paper

- Since that gives straight line which discribe \( Y = a + bX \)
- \( b = \beta \)
- \( a = -\frac{\alpha}{\beta} \ln(\eta) \)
- \( a = e^{-(\alpha/\beta)} \)

**Normal plot**

- Here plot \( Y = \Phi[\Phi^{-1}(F(t))] \) Vs \( T(i) \) on normal graph paper

- Since that gives straight line which discribe \( Y = a + bX \)
- \( \sigma = 1/b \)
- \( \mu = -a/b = \text{MTTF} \)

Thank you