Chapter 16 – Mechanisms of heat transfer
Mechanisms of heat transfer

• Heat can flow spontaneously from an object with a high temperature to an object with a lower temperature.

• Heat can only be transferred between objects, or areas within an object, with different temperatures (as given by the zeroth law of thermodynamics), and then, in the absence of work, only in the direction of the colder body (as per the second law of thermodynamics).

• There are 3 mechanisms of heat transfer
  – Conduction
  – Convection
  – Radiation
Conduction

• Conduction is the transfer of energy from the more energetic particle of a substance to the adjacent less energetic ones as a result of interactions between the particles.

• Conduction takes place in solids, liquids and gases.

• In liquids and gasses conduction heat transfer is due to the collisions and diffusion of the molecules during their random motion.

• In solids: it is due to the combination of vibrations of molecules in a lattice and energy transported by free electrons.
Rate of heat conduction

Consider steady heat conduction through a large plane wall,

- Area=$A$
- Thickness=$\Delta x$
- Temperature difference= $\Delta T=T_1-T_2$

Experiments have been shown that the heat transfer rate

$$\dot{Q}_{\text{cond}} \propto A$$

$$\dot{Q}_{\text{cond}} \propto T_1 - T_2 = \Delta T$$

$$\dot{Q}_{\text{cond}} \propto \frac{l}{\Delta x}$$

$$\dot{Q}_{\text{cond}} \propto A \frac{\Delta T}{\Delta x}$$

$$\dot{Q}_{\text{cond}} = -kA \frac{\Delta T}{\Delta x}$$
Fourier's law of heat conduction

\[ \dot{Q}_{\text{cond}} = -kA \frac{\Delta T}{\Delta x} \quad (W) \]

- Thermal conductivity (Wm\(^{-1}\)K\(^{-1}\)), measure of material's ability to conduct heat

\[ \frac{\Delta T}{\Delta x} \] - Temperature gradient, which has a negative slope

\[ \frac{\dot{Q}_{\text{cond}}}{A} = \dot{q} = -k \frac{\Delta T}{\Delta x} \quad (W/m^2) \]

- Rate of heat conduction per unit area
Simple example

\[
\frac{\dot{Q}_{\text{cond}}}{A} = \dot{q} = -k \frac{\Delta T}{\Delta x} \quad (W / m^2)
\]

\[
\dot{q} = 401 \times \frac{(30 - 20)}{1} \left[ \frac{W}{m \, ^oC} \times \frac{^oC}{m} \right] \\
= 4010 \, W / m^2
\]

\[
\dot{q} = 148 \times \frac{(30 - 20)}{1} \left[ \frac{W}{m \, ^oC} \times \frac{^oC}{m} \right] \\
= 1480 / m^2
\]

(a) Copper \((k = 401 \, W/m \cdot ^oC)\)

(b) Silicon \((k = 148 \, W/m \cdot ^oC)\)
Thermal conductors and thermal insulators

\[ \dot{Q}_{\text{cond}} = -kA \frac{\Delta T}{\Delta x} \quad (W) \]

To obtain high heat transfer, use materials with high thermal conductivity (thermal conductors; high \( k \)) and to reduce heat transfer use materials with low thermal conductivity (thermal insulators; low \( k \))
Thermal conductivity of some materials

• $k$ in most liquids is decreased with increasing temperature (water is exceptional)

• In gases and liquids $k$ decrease with increasing molar mass (in kinetic theory,

\[ k \propto \sqrt{\frac{T}{M}} \]

• In gases $k$ increases with increasing temperature
Some definitions in short

• \( C_p \ (\text{J/kg}) = \) Specific heat; material’s ability to store thermal energy (per unit mass).

• \( \rho C_p \ (\text{J/m}^3) = \) Heat capacity; material’s ability to store thermal energy (per unit volume).

• \( k \ (\text{W/m°C}) = \) Thermal conductivity; material’s ability to conducts heat.

• \( \alpha \ (\text{m}^2/\text{s}) = \) Thermal diffusivity; how fast heat diffuses through a material

\[
\alpha = \frac{\text{heat conducted}}{\text{heat stored}} = \frac{k}{\rho C_p}
\]
Measuring thermal conductivity of a material

Experiment: In a certain experiment, cylindrical samples of diameter 5 cm and length 10 cm are used. The two thermocouples in each sample are placed 3 cm apart. After initial transient, the electrical heater is observed to draw 0.4 A at 110 V, and both differential thermometers read a temperature difference of 15°C. Determine the thermal conductivity of the material.
Assumptions

• Steady state: temperature do not change.

• Insulated very well: heat loss through lateral surfaces are negligible.

• Entire heat generated by resistance heater is conducted through samples.

• The apparatus posses thermal symmetry.
\[ \dot{Q}_{\text{cond}} = -kA \frac{\Delta T}{\Delta x} \text{ (W)} \]

\[ k = \frac{\dot{Q}_{\text{cond}}}{A} \frac{\Delta x}{\Delta T} \]

\[ P_{\text{electrical power supply}} = VI \]

\[ \dot{Q}_{\text{cond}} = \frac{VI}{2} = \frac{110 \times 0.4}{2} [VA = W] = 22W \]

\[ A = \frac{1}{4} \pi D^2 = \frac{1}{4} \pi (0.05m)^2 = 0.00196m^2 \]

\[ k = \frac{(22W)(0.03m)}{(0.00196m^2)(15^\circ C)} = 22.4 W / m^\circ C \]
Conversion between SI and English units

- Heat, $1W = 3.41214 \text{ Btu/h}$
- Length, $1m = 3.2808 \text{ ft}$
- Temperature, $1^\circ \text{C} = 1.8^\circ \text{F}$
- Thermal conductivity, $1 \text{ W/m}^\circ \text{C} = 0.5778 \text{ Btu/h.ft.}^\circ \text{F}$
Convection

- Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion.

- The faster the fluid motion, greater the heat transfer

- It can be divided into two parts, natural convection and force convection

- Natural convection (free convection)
  - Fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid.

- Force convection
  - Fluid is forced flow over the surface by external means such as fan, pump
Force and natural (free) convection

- Forced convection
- Natural convection
- Natural convection
Newton’s law of cooling

\[ \dot{Q}_{\text{conv}} = hA_s (T_s - T_\infty) \quad (W) \]

\( h \) - convection heat transfer coefficient
Measuring convection heat transfer coefficient

Experiment: A 2m-long, 0.3cm-diameter electrical wire extends across a room at 15°C, as shown in figure. Heat is generated in the wire as a result of resistance heating, and the surface temperature of the wire is measured to be 152°C. Also, the voltage drop and electric current through the wire are measured to be 60V and 1.5A, respectively. Disregarding any heat transfer by radiation, determine the convection heat transfer coefficient for heat transfer between the outer surface of the wire and the air in the room.
- Assumptions
  - Steady state condition; no temperature change
  - Radiation heat transfer negligible.

\[ Q_{\text{conv}} = hA_s(T_s - T_{\infty}) \quad (W) \quad \Rightarrow \quad h = \frac{Q_{\text{conv}}}{A_s(T_s - T_{\infty})} \]

\[ A_s = \pi DL = \pi(0.003m)(2m) = 0.01885 m^2 \]

\[ Q_{\text{conv}} = P_{\text{electrical power supply}} = VI = (60V)(1.5A) = 90W \]

\[ h = \frac{Q_{\text{conv}}}{A_s(T_s - T_{\infty})} = \frac{90W}{(0.01885 m^2)(152 - 15) ^{\circ}C} = 34.9 W / m^2 ^{\circ}C \]
Radiation

- Radiation is the energy emitted by matter in the form of electromagnetic waves (or photons).

- It does not require a medium to transfer energy (this is how energy of the sun reaches the earth).

- It suffers no attenuation (weakening) in a vacuum. Radiation from the sun is attenuated by the Earth's atmosphere.

- Radiation is volumetric phenomenon: all solids, liquids gasses, absorb, emits or transmit radiation to varying degrees.

- However, radiation is usually considered to be a surface phenomenon for solids that are opaque to thermal radiation such as metals, wood and rocks since the radiation emitted by the interior regions of such material can never reach the surface, and radiation incident on such bodies is usually absorbed within a few microns from the surface.
The maximum rate of radiation that can be emitted from a surface at an absolute temperature of $T_s$ is given by the Stefan-Boltzmann law

$$\dot{Q}_{rad,\max} = \sigma A_s T_s^4$$

\[ Stefan \text{–} \ Boltsman \ Cons \ tan \ t, \ \sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4 \ (SI \ units) \]

or \[ 0.1714 \times 10^{-8} \text{ Btu/h.ft}^2\text{.K}^4 \]
Blackbody radiation

• In physics, a **black body** is an object that absorbs all electromagnetic radiation that falls onto it. No radiation passes through it and none is reflected. It is this lack of both transmission and reflection to which the name refers. These properties make **black bodies ideal sources** of thermal radiation.

• "Blackbody is a perfect absorber and perfect emitter.\[
\dot{Q}_{\text{rad,blackbody}} = \sigma A_s T_s^4
\]
The radiation emits by real bodies are less than black bodies

Radiation from a real body

\[ \dot{Q}_{\text{rad,real}} = \varepsilon\sigma A_s T_s^4 \]

\( \varepsilon \) is called emissivity of the surface: it is different to surface to surface, polish surfaces have low emissivities and rough and painted surfaces have high emissivities.
### Solar Absorptivity $\alpha_s$ and Emissivity $\varepsilon$ of Surfaces

<table>
<thead>
<tr>
<th>Surface</th>
<th>$\alpha_s$</th>
<th>$\varepsilon$ (300 K)</th>
<th>$\alpha_s/\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaporated aluminum film</td>
<td>0.09</td>
<td>0.03</td>
<td>3.0</td>
</tr>
<tr>
<td>Fused quartz on aluminum film</td>
<td>0.19</td>
<td>0.81</td>
<td>0.24</td>
</tr>
<tr>
<td>White paint on metallic substrate</td>
<td>0.21</td>
<td>0.96</td>
<td>0.22</td>
</tr>
<tr>
<td>Black paint on metallic substrate</td>
<td>0.97</td>
<td>0.97</td>
<td>1.0</td>
</tr>
<tr>
<td>Stainless steel, as received, dull</td>
<td>0.50</td>
<td>0.21</td>
<td>2.4</td>
</tr>
<tr>
<td>Red brick</td>
<td>0.63</td>
<td>0.93</td>
<td>0.68</td>
</tr>
<tr>
<td>Human skin (Caucasian)</td>
<td>0.62</td>
<td>0.97</td>
<td>0.64</td>
</tr>
<tr>
<td>Snow</td>
<td>0.28</td>
<td>0.97</td>
<td>0.29</td>
</tr>
<tr>
<td>Corn leaf</td>
<td>0.76</td>
<td>0.97</td>
<td>0.78</td>
</tr>
</tbody>
</table>
When surface of emissivity $\varepsilon$ and surface area of $A_s$ is completely enclosed by a much larger surface at absolute temperature of $T_{surr}$ separated by a gas (such as air) that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by,

$$\dot{Q}_{rad} = \varepsilon \sigma A_s (T_s^4 - T_{surr}^4)$$
Combine heat transfer coefficient

Radiation heat transfer to or from a surface surrounded by a gas such as air occurs parallel to conduction (or convection, if there is bulk gas motion) between the surface and the gas. Thus, the total heat transfer is determined by adding the contributions of both heat transfer mechanisms. By simplicity and convenience, this is often done by defining combined heat transfer coefficient $h_{combined}$ that includes the effects of both convection and radiation. The total heat transfer rate to or from a surface is by convection and radiation is expressed as,

$$
\dot{Q}_{total} = h_{combined} A_s (T_s - T_\infty)
$$
Example: The total heat transfer rate (by convection and radiation) of a surface is given by following expression. Derive and expression for the combined heat transfer coefficient ($h_{combined}$)

$$
\dot{Q}_{total} = h_{combined} A_s (T_s - T_\infty)
$$
\[ \dot{Q}_{\text{total}} = \dot{Q}_{\text{con}} + \dot{Q}_{\text{rad}} \]

Where:

\[ \dot{Q}_{\text{total}} = h_{\text{combined}} A_s (T_s - T_{\infty}) \]

\[ \dot{Q}_{\text{conv}} = h_{\text{con}} A_s (T_s - T_{\infty}) \]

\[ \dot{Q}_{\text{rad}} = \varepsilon \sigma A_s (T_s^4 - T_{\infty}^4) \]

\[ \dot{Q}_{\text{total}} = A_s h_{\text{con}} (T_s - T_{\infty}) + \varepsilon \sigma A_s (T_s^4 - T_{\infty}^4) \]

\[ \dot{Q}_{\text{total}} = A_s h_{\text{con}} (T_s - T_{\infty}) + \varepsilon \sigma A_s (T_s^2 + T_{\infty}^2)(T_s^2 - T_{\infty}^2) \]

\[ \dot{Q}_{\text{total}} = A_s h_{\text{con}} (T_s - T_{\infty}) + \varepsilon \sigma A_s (T_s^2 + T_{\infty}^2)(T_s + T_{\infty})(T_s - T_{\infty}) \]

\[ \dot{Q}_{\text{total}} = A_s \left( h_{\text{con}} + \varepsilon \sigma A_s (T_s^2 + T_{\infty}^2)(T_s + T_{\infty}) \right)(T_s - T_{\infty}) \]

\[ \dot{Q}_{\text{total}} = \left\{ h_{\text{con}} + \varepsilon \sigma A_s (T_s^2 + T_{\infty}^2)(T_s + T_{\infty}) \right\} A_s (T_s - T_{\infty}) \]

\[ \dot{Q}_{\text{total}} = h_{\text{combined}} A_s (T_s - T_{\infty}) \]

\[ h_{\text{combined}} = \left\{ h_{\text{con}} + \varepsilon \sigma A_s (T_s^2 + T_{\infty}^2)(T_s + T_{\infty}) \right\} \]
Total heat transfer from a body

Heat transfer takes place between objects with different temperatures and all three modes of heat transfer exists simultaneously. However, there are many situations when one mode dominates over others.

**Question:** How do we know which mode dominates over others?

Neglecting less important ones can usually simplify the calculation without significant error.
Simultaneous heat transfer mechanisms

- Opaque solids (inside): Heat transfer is only by conduction
- Semitransparent solids (inside): Heat transfer is by conduction and radiation
- Solid exposed to liquid or gas (with another surface): convection and radiation
- Still fluid: Conduction and possibly by radiation (no bulk fluid motion)
- Flowing fluid: Convection and radiation
- Vacuum: Heat transfer is only by radiation